

# Math 010 - Exam 1 Study Guide

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This study guide is designed to help you prepare specifically for Exam 1. Every item below corresponds directly to skills or ideas that appear on the exam versions.

## 1. Linear Systems and Consistency

What you should be able to do:

- Set up an augmented matrix for a system of equations.
- Determine specific conditions (in terms of variables like  $a$ ,  $b$ , and  $c$ ) that make a system consistent.
- Identify when a system is inconsistent using row reduction.

You should understand:

- The algebraic meaning of a "row of zeros" in the coefficient part vs. the augmented part.
- How to interpret the final row of a reduced matrix to find constraints.

## 2. Solutions as Linear Combinations

What you should be able to do:

- Read an augmented matrix in RREF and identify leading and free variables.
- Express the general solution set as a linear combination of numerical column vectors.

You should understand:

- How each free variable corresponds to a specific vector in the solution set.

## 3. Matrix Operations and Trace

What you should be able to do:

- Perform matrix addition, subtraction, and multiplication.
- Calculate the trace ( $\text{tr}$ ) of a square matrix.
- Identify when operations are undefined due to dimension mismatches.

You should understand:

- The properties of matrix multiplication (it is not always commutative).
- The definition of the trace as the sum of diagonal elements.

## 4. Inverses and the Inversion Algorithm

What you should be able to do:

- Compute the inverse of a  $2 \times 2$  matrix using the determinant formula.
- Solve a linear system  $Ax = b$  by using the inverse matrix  $A^{-1}$ .

- Use the inversion algorithm ( $[A \mid I] \rightarrow [I \mid A^{-1}]$ ) to determine if a  $3 \times 3$  matrix is invertible or singular.

You should understand:

- The conditions for a matrix to be singular (non-invertible).

## 5. Elementary Matrices and Symmetry

What you should be able to do:

- Find an elementary matrix  $E$  that performs a specific row operation to transform  $B$  into  $A$ .
- Determine values for unknowns to make a matrix symmetric ( $A = A^T$ ).

You should understand:

- The relationship between row operations and left-multiplication by elementary matrices.

## 6. Conceptual Facts and Proofs

What you should be able to do:

- Justify True/False statements about homogeneous systems and RREF.
- Construct logically sound proofs involving matrix properties (e.g., properties of transposes, inverses, or trivial solutions).

You should understand:

- Why homogeneous systems with more unknowns than equations always have nontrivial solutions.
- The algebraic properties of  $(AB)^{-1}$  and  $(A^T)^{-1}$ .

# Math 010: Linear Algebra Practice Worksheet

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## 1. Systems and Consistency

Determine all values of  $k$  for which the following system is consistent:

$$\begin{aligned}x + y + z &= 2 \\x + 2y + 4z &= k \\x + 4y + 10 &= k^2\end{aligned}$$

## 2. Solutions as Linear Combinations

The augmented matrix for a linear system has been reduced to:

$$\left[ \begin{array}{cccc|c} 1 & 4 & 0 & -2 & 1 \\ 0 & 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

Express the solution set as a linear combination of numerical column vectors, or explain why no solution exists.

## 3. Matrix Operations and Trace

Let  $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 0 & 5 \\ 1 & -1 & 3 \end{bmatrix}$

- Compute  $\text{tr}(AB)$ .
- Explain why  $A + B$  is undefined.

## 4. Inversion and Linear Systems

Let  $M = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ .

- Find  $M^{-1}$ .
- Use  $M^{-1}$  to solve

$$5x_1 + 3x_2 = 10$$

$$3x_1 + 2x_2 = 7$$

## 5. Elementary Matrices

Find an elementary matrix  $E$  such that  $EB = A$ , where  $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $A = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ .

## 6. Conceptual True/False

- True/False: If a  $3 \times 5$  homogeneous system has a solution other than the trivial solution, it must have infinitely many solutions. Justify.
- True/False: If  $A$  is an invertible matrix, then  $A^T$  must also be invertible. Justify.

Please review other T/F questions at the end of each homework assignment.

## 6. Proofs

Be able to prove statements such as:

- Prove that if  $A$  is an invertible square matrix, then the system  $A\mathbf{x} = \mathbf{0}$  has only the trivial solution.
- Show that if  $A^{-1}$  exists for an  $n \times n$  matrix, then it is unique.
- Prove that if  $A$  and  $B$  are invertible  $n \times n$  matrices, then  $AB$  is invertible and  $(AB)^{-1} = B^{-1}A^{-1}$ .
- Prove that if  $A$  is an invertible  $n \times n$  matrix, then  $A^T$  is also invertible and  $(A^T)^{-1} = (A^{-1})^T$ .
- Prove that if  $B$  is obtained from  $A$  by performing a sequence of elementary row operations, then there is a second sequence of elementary row operations, which when applied to  $B$  recovers  $A$ .
- Prove that if  $A$  is an invertible matrix and  $B$  is row equivalent to  $A$ , then  $B$  is also invertible.